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Some applications of Weak-multiplication modules in Neutrosophic set environment

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Abstract

Recently it has been proven that the multiplication module is a Weak-multiplication module $(W-MM)$. In this paper, we present some applications of $W-MM$ in neutrosophy theory. n-refined neutrosophic module M denoted by $M_n(I)$. We proved that $M_n(I)$ is regular and C.P.module. Also, every n-refined neutrosophic finitely generated ideal is a principal ideal of n-refined the ring R $(R_n(I))$. We proved that if $M_n(I)$ is multiplication module over ring $R_n(I)$ and $M_n(I)$ is of type S_2; then every f. generated $A_n(I)$ of $M_n(I)$ is n-refined multiplication module. Also, if $R_n(I)$ is n-refined neutrosophic ring, so $R_n(I)$ is principal ideal ring if and only if every n-refined neutrosophic multiplication module $M_n(I)$ is of type S. Finally, several results and applications have been presented in this paper with some new definitions, examples, and other properties.

Keywords: Multiplication module; Weak-multiplication module-Neutrosophic ring; Neutrosophic module; Regular ring.
2010 MSC: 13C05, 13C10, 16-XX, 20-XX

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1. Introduction

Module theory is a branch of abstract algebra that deals with the study of modules, which are generalizations of vector spaces over fields. Modules provide a framework for studying algebraic structures that share certain properties with vector spaces but do not necessarily have a field as their underlying structure. Some researchers define the module as a generalization of a vector space, where instead of working over a field, one works over a ring. A module consists of an abelian group together with a scalar multiplication operation defined by the ring. The ring can be a commutative ring with identity or a non-commutative ring.

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In module theory, the concept of multiplication modules [1] refers to modules that have an additional multiplication operation defined on their elements. The concept of faithful multiplication module [1] that is annihilator of M equal zero. This concept generalizes the notion of a module by incorporating a multiplicative structure in addition to the module's additive structure. This type of module has been studied by many researchers, including Anderson and Al-Shaniafi [2] studied multiplication modules with some properties of ideals. A generalization of multiplication modules offered by Perez et. al. [3]. Ansari-Toroghy and Farshadifar [4] obtained some related results regarding the concept of a comultiplication R-module. Azizi [5] characterized weak points of multiplication modules. Abed et al. [6] worked on several multitype modules like P-(S. P) Submodules [6], fractional module [7], classical Artinian module [8], injective module [9], CS-module [10], and other algebraic structures [11]. On the other hand, Smarandache [13] introduced the idea of a neutrosophic set as a generalization of the Zadeh model's Fuzzy set [14] as a way to represent and manipulate imprecise information. This idea came to address the many problems of life that the concept of the fuzzy set cannot deal with. This idea has gained the admiration of many researchers around the world, and therefore they have presented many research works aimed at solving the problems of daily life, including Khan et al. [15] works on an extended overview of neutrosophic set as well as several instances and extensions. Majumdar introduced an application to decision-making on neutrosophic sets. Al-Quran et al [17, 18] constructed some works on neutrosophic sets. Following them Al-sharqi et al [19, 20] studied several properties and structures of neutrosophic sets like real value [25], complex value [21], matrix value [22], and soft computing value [23] with their applications in real life [24]. Neutrosophic algebra, also known as neutrosophic set theory or neutrosophic logic, is a mathematical framework that extends fuzzy algebra theory and fuzzy algebra logic to handle uncertainty and vagueness in decision-making and reasoning. neutrosophic algebra has applications in various fields, including control systems, pattern recognition, expert systems, data analysis, decision-making, and artificial intelligence. It provides a mathematical framework to model and handle uncertainty and imprecision in these domains. neutrosophic algebra provides a rich mathematical framework to handle and reason with uncertain and imprecise information, allowing for more flexible and realistic modeling and decision-making. It has found wide applications in diverse fields where uncertainty and vagueness are inherent in the data or knowledge. Therefore, there is a lot of research work on this idea like Smarandache [25]. proposed neutrosophic triplets to introduce the innovative notion of neutrosophic triplet group. Jun et al. [26] considered the idea of a neutrosophic quadruple BCK/BCI-number. Abed et al. [27, 28] introduced some neutrosophic algebraic structures, and module theory and studied their properties and applications. Olgun and Bal [29] give some studies on neutrosophic module theory. Auad et al. [30, 31] introduced some results of best co-approximation specifications of co-proximal and co-Chebyshev of unbounded functions in weighted spaces.
We will present a follow-up to these works in this work we will give some applications of the W-multiplication module in Neutrosophy theory. n-refined Neutrosophic module M denoted by $M_n(I)$. We proved that $M_n(I)$ is n-refined Neutrosophic regular and n-refined Neutrosophic C.P. module. Also, every n-refined Neutrosophic finitely generated ideal is a principal ideal of n-refined ring R $(R_n(I))$. Finally, several results and applications have been presented in this paper with some new definitions, examples, and other properties. Also, we should be notes all rings in this paper are commutative with 1 and all modules over the ring R are unital. In this paper, we presented several basic concepts which useful later. Also, we introduced new results about neutrosophic weak-multiplication module.

2. Preliminaries

Definition 2.1. *[6] A ring R is a commutative with 1. If the following are holds $R \times M \to M(a,b) \to ab$ such that M is a commutative group:*

1. $(ab_1)b = a(b_1b)$
2. $(a_1 + a_2)b = a_1b + a_2b$
3. $a(b_1 + b_2) = ab_1 + ab_2$
4. $1 \cdot b = b = b \cdot 1,$

then M is called R-module.

Definition 2.2. *[6] A subset M_1 is said to be submodule of $M\,(M_1 \leq M)$ if closed with $(+)$ and scalar multiplication:*

1. $a + b \in M_1, \forall a, b \in M_1$.
2. $ab \in M_1, \forall a \in R, b \in M_1$.

Definition 2.3. *[13] A neutrosophic S is defined as $K = \{(\zeta, T_k(\zeta), I_k(\zeta), F_k(\zeta)) : \zeta \in S\} \ni t_k, i_k, f_k : S \to [0, 1]$.*

Definition 2.4. *[29] Let $(R, +, \cdot)$ be a ring and a neutrosophic set denoted by R and I. So Neutrosophic $R = (R(I), +, \cdot)$ is called a neutrosophic ring.*

Definition 2.5. *[29] Let $(M, +, \cdot)$ be a module over R. Then $(M(I), +, \cdot)$ is a weak neutrosophic module over R, and it is called a strong neutrosophic module if it is a module over $R(I)$.*

Definition 2.6. *[28] Let $P = \{(T_p(\eta), I_p(\eta), F_p(\eta)) : \eta \in R\}$. Then P is a neutrosophic ideal if $\forall \eta, \theta \in R$ be a neutrosophic of module over neutrosophic R. Then any neutrosophic subset neutrosophic K of neutrosophic M is neutrosophic submodule:*

1. $T_P(\eta - \theta) \geq T_P(\eta) \wedge T_P(\theta)$.
2. $I_P(\eta - \theta) \geq I_P(\eta) \wedge I_P(\theta)$.
3. $F_P(\eta - \theta) \leq F_P(\eta) \vee F_P(\theta)$.
4. $T_P(\eta\theta) \geq T_P(\eta) \vee T_P(\theta)$.
5. $I_P(\eta\theta) \geq I_P(\eta) \vee I_P(\theta)$.
6. $F_P(\eta\theta) \leq F_P(\eta) \wedge F_P(\theta)$.

Definition 2.7. *[1] Let K be a field and A be an algebra over K (A not neccssarily commutative algebra). Let M be an A-module and it's called a faithful multiplication A-module if $Ann_A(M) = 0$.*

3. Main results

In this part, we will discuss some result and some application in weak-multiplication module in neutrosophic set environment. Also, we will support these results with some examples.

Definition 3.1. *Let $M(I)$ be a indeterminacy w-multiplication module over neutrosophic ring $R(I)$, we say $M_n(I)$ is n-refined neutrosophic w-multiplication module (of type S_p) if for p-generated n-refined neutrosophic submodule $A_n(I)$ there exists n-refined neutrosophic mapping $g : M_n(I) \to A_n(I)$.*

Remark 3.2. *We notice that:*

(i) *$M_n(I)$ is n-refined neutresophic module is type of (S) if each n-refined neutrosophic submodule $A_n(I)$ of $M_n(I)$ is an epimorphic image of $M_n(I)$.*

(ii) *Every n-refined neutresophic semisimple module is type of s, because every n-refined neutr-sophic semi-simple module has n-refined neutrosophic direct summand of submodules.*

Definition 3.3. *Any neutrosophic module $M_n(I)$ is called n-refined cyclic neutrosophic module $M_n(I)$ if for all $mI \in M_n(I)$, so $M_n(I) = R_n(I)mI$.*

Definition 3.4. *Any module $M_n(I)$ is called C.P. module if every cyclic submodule $A_n(I)$ is projective with $M_n(I)$ is n-refined, neatrosephic finitely generated projective module of every n-refined neutrosophic f. generated submodule is projective.*

Recall that $M_n(I)$ is called $Z_n(I)$ neutrosophic regular is every n-refined neutrosophic $(M = Rx)$ Submodule (n-refined neutrosophic f.g module) is Projective and direct summand.

Theorem 3.5. *Let $M_n(I)$ be module over $R_n(I)$. Then the following statements are equivalent.*

1. *$M_n(I)$ is $Z_n(I)$ regular.*
2. *$M_n(I)$ FGP- module.*
3. *$M_n(I)$ is C.P. Module.*

Proof. $1 \implies 2$. Let $A_n(I)$ be n-refined neutrosophic f. generated submodule of $M_n(I)$. So $A_n(I)$ is a direct summand of $M_n(I)$. i.e. $M_n(I) = \oplus A_n(I)$. Therefore $A_n(I)$ is projective. Hence $A_n(I)$ is an epimorphic image of $M_n(I)$.
$2 \implies 3$. Clear.
$3 \implies 1$. Assume that $A_n(I)$ is cycle submodule of $M_n(I)$.Then there exists an epimerphism $g : M_n(I) \to A_n(I)$.
Note that $A_n(I)$ is projective, so there is a sequence is n-refined split. Hence $A_n(I)$ is a direct summand. Thus $M_n(I)$ is Z_n neutrosophic regular. □

Definition 3.6. *Let $M_n(I)$ be module over $R_n(I)$ and let $A_n(I)$ be n-refined neutrosophic submodule of $M_n(I)$. Then $A_n(I)$ is called pure submodule of $M_n(I)$, if $A_n(I) \cap M_n(I)J_n(I) = J_n(I)A_n(I)\,\forall J_n(I)$ is n-refined neutrosophic ideal of $R_n(I)$.*

Definition 3.7. *Any $M_n(I)$ is called F-regular if $\forall A_n(I) \leq M_n(I)$ is n-refined neutrosophic pure.*

Recall that $A_n(I) \leq M_n(I)$ is called n-rfined neutrosophic strongly pure if for each n-refined neutrosophic finite sub module, $A_n^{'}(I)$ of $A_n(I)$, there exists n-refined neutrosophic homomorphism $g : M_n(I) \to A_n(I)$ such that $f(aI) = aI$, $aI \in A_n(I)$.

Remark 3.8. *In the following:*

(i) $M_n(I)$ is n-refined strongly F. regular if every n-refined neutresuphic submodule $A_n(I)$ of $M_n(I)$ is n-refined neutrosophic strongly pure [7].

(ii) Every n-refined neutresaphic strongly F. regular module is n-refined neutrosophic F. regular.

(iii) $Z_n(I)$ regular $\implies$ F.regular .

Proposition 3.9. *Let $R_n(I)$ be ring. Then every finitely generated ideal is principal.*

Proof. Let $J_n(I)$ be n-refined finitely generated neutrosoPhic ideal of $R_n(I)$. Then $\exists g : R_n(I) \to J_n(I)$ is an epimorphism. so, we know that $R_n(I)$ is cyclic module. Hence $J_n(I)$ is principal. □

Corollary 3.10. *Let $M_n(I)$ be module other ring. Suppose that $M_n(I)$ such that $ann(a_0I) = ann(M_n(I))$. If $g : M_n(I) \to R_n(a_0I)$ is an epimorphism, so $M_n(I)$ is of type S_∞.*

Proof. $\forall aI \in M_n(I)$, we deline a mapping $f_{aI} : R_n(a_0I) \to R_n(aI)$ by $f_{aI}(bIa_0I) = (bI)(I)$. then f is well–define and hence $ba_0I = \phi$, so $bI \in ann(a_0I) = ann(M_n(I))$. Thus $bIaI = \phi$. Hence we get $M_n(I)$ is of type S_1 and then is of type S_∞. □

Proposition 3.11. *Let $M_n(I) = M(I_1, I_2, \ldots I_n)$ be module over p.p. ring $R_n(I)$. If $M_n(I)$ is projective module (n-refined c.p. module), then $R_n(I) \oplus M_n(I)$ is n-refined z_n-regular module.*

Proof. Let $M_n(I)$ be projective module. Hence $R_n(I) \oplus M_n(I)$ is n-refined projective module. But $R_n(I)$ is n-refined p.p. ring. then $R_n(I) \oplus M_n(I)$ is n-refined c.p. module. Also, $R_n(I) \oplus M_n(I)$ is n-refined of type S_∞. So $M_n(I)$ is n-refined z_n-regular. If $M_n(I)$ is n-refined C.P. module, so, $R_n(I) \oplus M_n(I)$ is n-refined C.P. module. On the other hand, if $R_n(I) \oplus M_n(I)$ is of type S, hence $R_n(I) \oplus M_n(I)$ is n-refined z_n-regular module. □

Proposition 3.12. *Let $R_n(I)$ ben n-refined ring. So $R_n(I)$ is principal ideal ring if and only if every n-refined neutrosophic multiplication module $M_n(I)$ is of type S.*

Proof. Assume that $R_n(I)$ is P.I.R. with $M_n(I)$ is n-refined multiplication module. Suppose that $A_n(I)$ be submodule of $M_n(I)$. Then there exists n-refined ideal $J_n(I)$ of $R_n(I) \ni A_n(I) = J_n(I)M_n(I)$. So $\exists aI \in R_n(I) \ni J_n(I) = R_n(I)(aI) \wedge A_n(I) = (aI)M_n(I)$. Now we define $f_B : M_n(I) \to A_n(I)$ by $f(bI) = (aI)(bI)$, $bI \in M_n(I)$. Then f_B is $R_n(I)$-epimorphism. Conversely, $R_n(I)$ is n-refined neutrosophic multiplication module. Hence $R_n(I)$ is P.I.R. □

Theorem 3.13. *Let $R_n(I)$ be ring. Then if $R_n(I)$ is Bezout ring, any multiplication neutrosophic module is of type S_∞.*

Proof. Assume that $R_n(I)$ is n-refined Bezout neutrosophic ring with $M_n(I)$ is multiplication module. If $A_n(I)$ is finitely generated submodule of $M_n(I)$, so $\exists J_n(I)$ is n-refined ideal of $R_n(I)$. Such that $A_n(I) = J_n(I)M_n(I)$. Hence may be close $J_n(I)$ to finitely generated. Therefore, $\exists aI \in R_n(I) \ni J_n(I) = R_n(I)(aI)$ and $A_n(I) = J_n(I)M_n(I)$. Thus $f_\infty : M_n(I) \to A_n(I)$ and hence $M_n(I)$ is of type S_∞. □

Corollary 3.14. *Let $M_n(I)$ multiplication module over ring $R_n(I)$. If $M_n(I)$ is of type S_2; then every f. generated $A_n(I)$ of $M_n(I)$ is n-refined multiplication module.*

Proof. Let any 2-generated submodule $A_n(I)$ of $M_n(I)$. so $f : M_n(I) \to A_n(I)$ is epimorphism. Hence $A_n(I)$ is multiplication module. Therefore $A_n(I)$ is locally cyclic. Suppose that $K_n(I)$ generated by (K_1I, K_2I, K_3I) inside $M_n(I)$. So $K_n(I) = R_n(I)(K_1I)+R_n(I)(K_2I)+R_n(I)(K_3I)$. Hence k is 2-generated. This means for each $J_n(I)$ prime ideal of $R_n(I)$. There exists 2-generated submodule $A_n(I)$ of $M_n(I)$ such that $A_{n_p}(I) = k_{n_p}(I)$. We have $A_n(I)$ is multiplication module. So $A_{n_p}(I) = k_{n_p}(I)$ is n-refined cyclic submodule. Therefore $K_n(I)$ is multiplication module. □

Proposition 3.15. *Let $M_n(I)$ be faithful multiplication module over ring $R_n(I)$. If $R_n(I)$ is p.p. ring with $M_n(I)$ is of S_1 type, so $M_n(I)$ is a $Z_n(I)$-regular .*

Corollary 3.16. *Let $M_n(I)$ be faithful multiplication module over p.p. ring $M_n(R)$. If $End(M_n(I))$ is regular, then $M_n(I)$ is of type of S_∞.*

Proposition 3.17. *Let $M_n(I)$ be multiplication module, $N_n(I)$ be n-refined submodule of $M_n(I)$. If $M_n(I)$ has type S_∞, then $\frac{M_n(I)}{N_n(I)}$ has type S_∞.*

Proof. Suppose that $K_n(I)$ be n-refined submodule of $M_n(I)$ with $N_n(I) \leq M_n(I)$. So assume that $\frac{k_n(I)}{N_n(I)} \leq \frac{M_n(I)}{N_n(I)}$ is n-refined faithful generated, $k_n(I)$ is also faithful generated submodule of $M_n(I)$. But $M_n(I)$ has type S_∞, so $\exists f_n(I) : M_n(I) \to K_n(I)$. Also $N_n(I)$ is n-refined invaiant subvmodule of $M_n(I)$. Hence $f_n(I)(N_n(I)) \subseteq N_n(I)$. Thus $f_n(I)$ induces epi. $f_n(I) : \frac{M_n(I)}{N_n(I)} \to \frac{k_n(I)}{N_n(I)}$. □

Theorem 3.18. *Let $M_n(I)$ be module has type S_∞. So $M_{np(I)}(I)$ has type S_∞ where $P_n(I)$ is n-refined prime ideal of $R_n(I)$.*

Proof. Suppose that $P_n(I)$ is n-refined prime ideal of $R_n(I)$. Assume that $N_n(I) \leq M_{np(I)}(I)$ Hence $\exists K_n(I) \leq M_n(I), k_{n_{P_n(I)}}(I) = N_n(I)$. So $K_n(I)$ n-refined finitely generated when $N_n(I)$ is also n-refined finitely generated submodule of $M_n(I)$.
$\exists f_n(I) : M_n(I) \to k_n(I) \ni f_n(I)$ induce $R_{n_{P_n(I)}}(I)$-homomorphism $f_{n_{P_n(I)}}(I) : M_{n_{P_n(I)}}(I) \to K_{n_{P_n(I)}}(I)$ by: $f_{n_{P_n(I)}}(I)(\frac{mI}{tI}) = \frac{f_n(I)(m)}{tI}$. Thus $f_{n_{P_n(I)}}(I)$ is an epimorphism. □

Remark 3.19. *Note that the opposite of theorem 3.18 is not true. Let $R_n(I)$ be regular ring and not semisimple ring. Hence $R_{n_{P_n(I)}}(I)$ is field, where $P_n(I)$ is prime ideal of $R_n(I)$. Therefore $R_{n_{P_n(I)}}(I)$ is as $R_n(I)$-module has type S_∞. Thus $R_n(I)$ has no type S, it has type S_∞.*

Theorem 3.20. *Let $M_n^{'}(I)$ and $M_n^{''}(I)$ be two modules have type S. If $M_n(I) = M_n^{'}(I) \oplus M_n^{''}(I)$ is direct sum, then $M_n(I)$ has type S_n.*

Proof. Suppose that $N_n(I) \leq M_n(I)$. So $N_n(I) = N_n^{'}(I) \oplus N_n^{''}(I)$; $N_n^{'}(I) \leq M_n(I), N_n^{''}(I) \leq M_n(I)$. $N_n(I)$ is finitely n-generated. So $N_n^{'}(I), N_n^{''}(I)$ are n-refined n-generated. So $\exists f_n^{'}(I), f_n^{''}(I) \ni f_n(I) : M_n(I) \to N_n(I)$. Hence $f_n^{'}(I) \oplus f_n^{''}(I)$ is epi.from $M_n^{'}(I) \oplus M_n^{''}(I) \to N_n^{'}(I) \oplus N_n^{''}(I)$. □

Corollary 3.21. *Let $M_n(I) = M_n^{'}(I) \oplus M_n^{''}(I)$ where $M_n^{'}(I)$ and $M_n^{''}(I)$ are $R_n(I)$-modules and $M_n(I)$ has type $S(S_\infty)$. If $Hom(M_n^{'}(I), M_n^{''}(I)) = \phi$, then $M_n^{'}(I)$ has type $S(S_\infty)$.*

Proof. Suppose that $N_n^{'}(I) \leq M_n^{'}(I)$. Hence $N_n^{'}(I) \leq M_n(I)$ is finitely generated submodule of $M_n(I)$ and has type (S_∞). Then $\exists f_n(I) : M_n(I) \to N_n(I)$. Suppose that $f_n^{'}(I) = \frac{f_n(I)}{M_n(I)}$. But $Hom(M_n^{'}(I), M_n^{''}(I)) = \phi$, so $f_n(I)(M_n^{'}(I)) \subseteq M_n(I)$. Hence $f_n(I)(M_n^{'}(I)) = N_n^{'}(I)$. Thus $M_n^{'}(I)$ has type $S(S_\infty)$. □

4. Conclusion

In this study, we have developed cyclic, multiplication,and finitely generated modules using the concept of the neutrosophic set. Additionally, we displayed the recently proposed relations for neutrosophic weak-multiplication modules along with a few instances and notes. Let $R_n(I)$ be ring. Then each n-refined finitely generated neutrosophic ideal is principal. Furthermore, suppose that $M_n(I)$ is an n-refined faithful multiplication neutrosophic module over ring $R_n(I)$. If $R_n(I)$ is p.p. ring with $M_n(I)$ is of S_1 type, so $M_n(I)$ is a $Z_n(I)$-regular. Finally, we presented some examples, remarks and properties about this work in order to study algebraic structures.

Conflicts of Interest: "The authors declare no conflict of interest."

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