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Solving (3+1) D- New Hirota Bilinear Equation Using Tanh Method and New Modification of Extended Tanh Method

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Abstract

In this article Tanh method is considered as effective approach to get solutions of some types of non-linear partial differential equations. Then we suggested new modification for extended Tanh method as highly effective approach to obtaining precise traveling wave solutions to these types of equations. Then using both methods, to solve the (3+1) D- new Hirote bilinear equation (NHBE) and then we compare between the results to illustrate the effectiveness of suggested modification. The interpretation of these solutions presented graphical to illustrate behaviors of the solutions gives us some popular shapes include solutions of type solitary wave, the singular wave, the kink wave, and the singular kink wave.

Keywords: Nonlinear PDEs; Hirote bilinear equation (HBE); Tanh method; Extended tanh method.
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1. Introduction

Non-linear hyperbolic equation, wave model play an important role in many fields such as scientific and engineering such ocean engineering, materials science, chemical physics, chemistry, biology, solid state physics, fiber optics, chemical kinetics, oceanography, signal processing, deep water waves, and various from other non-linear disciplines [1].

Obtaining precise or numerical solutions of nonlinear PDEs holds significant importance across various scientific and engineering disciplines. Nonlinear phenomena hold significant importance across various scientific disciplines, notably in plasma waves, solid-state physics, fluid, plasma physics, mechanics, and chemical

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physics. The investigation of precise and numerical solutions, particularly those pertaining to travelling wave phenomena in nonlinear equations with in the field of mathematical physics, holds significant in soliton theory [2, 3].

Many reliable methods have been discovered or developed to find the precise solutions of non-linear problems, among them the Hirota bilinear theory [4], Darboux transformation [5], the VIM [6, 7], ADM [8, 9], the HPM [10, 11, 12], parameter expansion method [13, 14, 15, 16], HAM [17, 18, 19, 20, 21, 22], spectral collocation method [23, 24, 25, 26, 27], and the Exp-function method [28, 29, 30, 31, 32, 33].

New Hirota bilinear equation (NHBE) has applications in the study of physics specially plasma physics, astrophysics, turbulent flows, fusion energy and other nonlinear phenomena. Nowadays, these applications are very interesting topics in the modern research. So, we need to deep investigation and study of these physical problems, and then we need to find their exact analytical solutions.

The one dimensional space Korteweg–de Vries (KdV) equation has the form:

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1}$$

Which is a nonlinear evolution equation (NLEE) used as model describe the behavior of waves in shallow water. Other nonlinear evolution equation that describes the evolution of small amplitudes for long waves and the Kadomtsev-Petviashvili equation (KPE) which is progressive dependence on the transverse coordinate. This equation is characterized by two spatial variables and one temporal variable. The KP problem has the following form:

$$(u_t + 6uu_x + u_{xxx})_x + 3\vartheta^2 u_{yy} = 0 \tag{2}$$

The high-dimensional and extended version of the KdV problem is one of such NLEEs mentioned above lines has the following form as (3 + 1) D- new Hirota bilinear equation (NHBE) [34, 35, 36].

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{zz} = 0 \tag{3}$$

The main aim of this article is to derive precise solutions of type travelling wave for Eq. (3).

The organization of this article is as follows: In Section 2, the Tanh method has been discussed and then used to solve Eq. (3). In Section 3, the modified extended Tanh method has been discussed and then applied to solve Eq. (3). Graphical representation for solution we have shown in Section 4. Section 5 consist Discussion of the results. Conclusions are given final Section.

2. Tanh method

This section consist a short review of the tanh method which was described by Malfliet in [37] and Wazwaz [38, 39, 40]. First of all, let us consider the following non-linear PDEs:

$$P\left(u, u_x, u_y, u_z, u_t, u_{xx}, u_{yy}, u_{zz}, u_{xy}, \ldots\right) = 0 \tag{4}$$

Where P is polynomial of the dependent changeable u and its derivatives. If we suppose $f(\xi) = u(x, y, z, t)$,where $\xi = k.\,(x + y + z - ct\ )$ or $\xi = k.(x + y + z + ct)$.

We can take the following changes:

$$\frac{d}{dt} = \pm ck\frac{d}{d\xi}, \frac{d}{dx} = k\frac{d}{d\xi}, \frac{d}{dy} = k\frac{d}{d\xi}, \frac{d}{dz} = k\frac{d}{d\xi}, \frac{d^2}{dx^2} = k^2\frac{d^2}{d\xi^2}, \frac{d^3}{dx^3} = k^3\frac{d^3}{d\xi^3}$$

And so on, then Eq. (4) transformed from PDE to ODE say:

$$F\left(f, f', f'', f''', \ldots\right) = 0 \tag{5}$$

Integration Eq. (5). Suppose that the constant of integration is zero, for simplicity.

We present a new independent variable for the tanh method.

$$Y\left(x,y,z,t\right)=\tanh(\xi) \tag{6}$$

which leads to the change of variables as follow:

$$\frac{d}{d\xi}=(1-Y^2)\frac{d}{dY}$$

$$\frac{d^2}{d\xi^2}=-2Y\left(1-Y^2\right)\frac{d}{dY}+(1-Y^2)^2\frac{d^2}{dY^2}$$

$$\frac{d^3}{d\xi^3}=2\left(1-Y^2\right)\left(3Y^2-1\right)\frac{d}{dY}-6(1-Y^2)^2\frac{d^2}{dY^2}+(1-Y^2)^3\frac{d^3}{dY^3} \tag{7}$$

Therefore, the solution is expressed as a finite series expansion:

$$u\left(x,y,z,t\right)=f\left(\xi\right)=\sum_{i=0}^{m}a_iY^i \tag{8}$$

Where $k, c, a_0, a_1, a_2, \ldots$ have to be calculated and m can be calculated by balancing the linear highest order term in Eq. (5) with the nonlinear terms. Substituting Eq. (8) into Eq. (5) we get the result in a series of algebraic equations for $k, c, a_0, a_1, a_2, \ldots$.

We have relations obtained by vanish all coefficients of Y^i. So: $k, c, a_0, a_1, a_2, \ldots$ can be obtained. Finally using Eq. (8) to get analytic solution $u\left(x,y,z,t\right)$.

2.1. *Application*

Herein, we use tanh method to get the accurate solution for the NHBE that is Eq. (3).

$$\text{Let } f\left(\xi\right)=u\left(x,y,z,t\right),\ \text{ where } \xi=k.(x+y+z+ct) \tag{9}$$

Substituting Eq. (9) into Eq. (3), we have ODE:

$$ck^2f''-k^4f^{(4)}-3k(k^2f'*f')'-3k^2f''+3k^2f''=0$$

$$ck^2f''-k^4f^{(4)}-3k(k^2f'*f')'=0 \tag{10}$$

Integrating Eq. (10) gives

$$ck^2f'-k^4f'''-3k^3(f')^2=0 \tag{11}$$

We assumed that the constant of integration is zero.

The higher order derivative component f''' is balancing with the nonlinear term $(f')^2$ in Eq. (11), we get the value M = 1, and as a result,

$$u\left(x,y,z,t\right)=f\left(\xi\right)=\sum_{i=0}^{m=1}a_iY^i=a_0+a_1Y \tag{12}$$

The Eq. (11) reduces to

$$ck^2\left[\left(1-Y^2\right)\frac{df}{dY}\right]-k^4[2\left(1-Y^2\right)\left(3Y^2-1\right)\frac{df}{dY}-6(1-Y^2)^2\frac{d^2f}{dY^2}+(1-Y^2)^3\frac{d^3f}{dY^3}]-3k^3[(1-Y^2)\frac{df}{dY}]^2=0 \tag{13}$$

Substituting f', f''' from Eq. (12) into Eq. (13) produces the set of algebraic equations for the variables a_0, a_1, c, k, that is

$$Y^0 : ck^2a_1 + 2k^4a_1 - 3k^3a_1^2 = 0$$
$$Y^2 : -ck^2a_1 - 6k^4a_1 - 2k^4a_1 + 6k^3a_1^2 = 0$$
$$Y^4 : 6\ k^4a_1 - 3\ k^3a_1^2 = 0$$

We have $a_1 = 2k, c = 4k^2, k = k$
If $k = 1 \rightarrow a_1 = 2$ and $c = 4$. So

$$u\,(x, y, z, t) = f\,(\xi) = \sum_{i=0}^{m=1} a_iY^i = a_0 + a_1Y = a_0 + 2\tanh\,(x + y + z + 4t)\,.$$

Since $Y = \tanh(\xi)$.

$$\text{In particular if } a_0 = 0\ \Rightarrow u\,(x, y, z, t) = 2\tanh\,(x + y + z + 4t) \tag{14}$$

3. Suggested modification of extended Tanh method

Consider the following general nonlinear PDE:

$$P\,(u, u_x, u_y, u_z, u_t, u_{xx}, u_{yy}, u_{zz}, u_{xy}, \ldots) = 0 \tag{15}$$

Where $u(x, y, z, t)$ indicate an unknown function to be found and P is polynomial in $u(x, y, z, t)$
Step 1: We use a compound variable ξ to combine the real variables x, y, z with t as follow:

$$f\,(\xi) = u\,(x, y, z, t)\,, \text{ where } \xi = k.(x + y + z \pm ct) \tag{16}$$

Where c is the parameter representing speed of the wave. Eq. (15) is transferred into an ODE for $f(\xi)$ using Eq. (16), so

$$F\left(f, f', f'', f''', \ldots\right) = 0 \tag{17}$$

Where the $'$ denote the ordinary derivatives with respect to ξ, and F is a polynomial of u and its derivatives.
Step 2: Integration Eq. (17). For simplicity, suppose that the constant of integration is zero.
Step 3: The solution of type traveling wave for Eq. (16) can be expressed as the form:

$$f\,(\xi) = a_0 + \sum_{i=1}^{m} \left(a_iY^i + b_iY^{-i}\right) \tag{18}$$

and

$$Y = \tanh(\xi) \tag{19}$$

Where $a_0, a_i, b_i;\ i = 0, 1, 2, \ldots, r$ arbitrary constant can be determined later and Y satisfies the equation

$$Y' = 1 - Y^2 \tag{20}$$

Step 4: To find the positive integer m, the homogeneous equilibrium between the derivatives with highest-order in Eq. (17) and the highest-order nonlinear terms is used.
Step 5: By substituting Eq. (18) and Eq. (20) into Eq. (17) with the value m where calculated in step 4, and by accumulating all the expressions of the same power $Y^i, i = 0, \pm1, \pm2, \ldots$ and equating them to zero, we get an algebraic system can be solved by Maple to get the values of a_0, a_i, b_i, k and c.
Step 6: Substituting the values in above steps and $Y = \tanh(\xi)$ into Eq. (18) to get the exact solution of Eq. (15).

3.1. Application

In this subsection we use suggested modification of extended tanh method to get exact solution of Eq. (3).

$$\text{Let } f(\xi) = u(x, y, z, t), \text{ where } \xi = k.(x + y + z + ct) \tag{21}$$

Substituting Eq. (21) into Eq. (3), to get the following ODE

$$ck^2 f'' - k^4 f^{(4)} - 3k(k^2 f' * f')' - 3k^2 f'' + 3k^2 f'' = 0$$

$$ck^2 f'' - k^4 f^{(4)} - 3k(k^2 f' * f')' = 0 \tag{22}$$

Integrating Eq. (22) gives

$$cf' - k^2 f''' - 3k(f')^2 = 0 \tag{23}$$

We assumed that the constant of integration is zero.

By using step (4) in section 3, balancing $(f')^2, f'''$, gives $m = 1$. Hence:

$$f(\xi) = a_0 + \sum_{i=1}^{1} (a_i Y^i + b_i Y^{-i})$$

$$f(\xi) = a_0 + a_1 Y + b_1 Y^{-1} \tag{24}$$

From Eqs. (20) and (24), we obtain:

$$f' = a_1 + b_1 - a_1 Y^2 - b_1 Y^{-2} \tag{25}$$

$$f''' = -2a_1 + 2a_1 Y^2 + 6a_1 Y^2 - 6a_1 Y^4 - 6b_1 Y^{-4} + 6b_1 Y^{-2} + 2b_1 Y^{-2} - 2b_1 \tag{26}$$

$$(f')^2 = a_1^2 Y^4 - 2a_1^2 Y^2 + a_1^2 - 2a_1 b_1 Y^2 - 2a_1 b_1 Y^{-2} + 4a_1 b_1 + b_1^2 Y^{-4} - 2b_1^2 Y^{-2} + b_1^2 \tag{27}$$

Substituting Eqs. (25), (26) and (27) into Eq. (23), we have:

$$\begin{aligned}
&Y^0 : ca_1 + cb_1 + 2k^2 a_1 + 2k^2 b_1 - 3ka_1^2 - 12ka_1 b_1 - 3kb_1^2 = 0 \\
&Y^2 : -ca_1 - 2k^2 a_1 - 6k^2 a_1 + 6ka_1^2 + 6ka_1 b_1 = 0 \\
&Y^4 : 6k^2 a_1 - 3ka_1^2 = 0 \\
&Y^{-2} : -cb_1 - 6k^2 b_1 - 2k^2 b_1 + 6ka_1 b_1 + 6kb_1^2 = 0 \\
&Y^{-4} : 6k^2 b_1 - 3kb_1^2 = 0
\end{aligned}$$

Using the Maple, we get the following results.

Case 1: $c = 4k^2, b_1 = 0, a_1 = 2k, k = k$ Then:

$$u_1(x, y, z, t) = f(\xi) = a_0 + 2k \tanh\left(k\left(x + y + z + 4k^2 t\right)\right)$$

Case 2: $c = 4k^2, a_1 = 0, b_1 = 2k, k = k$ Then:

$$u_2(x, y, z, t) = f(\xi) = a_0 + 2k coth\left(k\left(x + y + z + 4k^2 t\right)\right)$$

Case 3: $c = 16k^2, a_1 = 2k, b_1 = 2k, k = k$

$$u_3(x, y, z, t) = a_0 + 2k\ \tanh\left(k\left(x + y + z + 16k^2 t\right)\right) + 2k \coth\left(k(x + y + z + 16k^2 t)\right)$$

From case 1, we see that: If $a_0 = 0, k = 1$ we get:

$$u_1(x, y, z, t) = 2 \tanh(x + y + z + 4t) \tag{28}$$

From case 2, we see that: If $a_0 = 0, k = 1$ we get

$$u_2(x, y, z, t) = 2 \coth(x + y + z + 4t) \tag{29}$$

4. Graphical representation of solution

In this section, we have plotted obtained solutions of type traveling wave by software Maple of the NHBE For a clearer explanation:

The solution denoted by Eq. (28) is solution of type kink. Figure 1, illustrate the shape of the exact solution of the (3+1) D- NHBE presented by Eq. (3) (only the case with $a_0 = 0, k = 1, y = 1, z = 0, -10 \leq x \leq 10, t \leq 10$) and other kink wave shape appears in Figure 2a, which illustrate the shape of the exact solution for Eq. (3) when $a_0 = 0, k = 1, -10 \leq y \leq 10, z = 0, -10 \leq x \leq 10, t = 0$. Also Another kink wave shape appears in Figure 2b, Shows the shape of the exact solution of the Eq. (3) with $a_0 = 0, k = 1, -10 \leq y \leq 10, z = 0, -10 \leq x \leq 10, t = 0.5$.

The solution denoted by Eq. (29) is named the solution of type singular kink and presented in Figure 3. We can clearly see that the shape of the exact solution of type singular kink for the Eq. (3) with $a_0 = 0, k = 1, -10 \leq y \leq 10, z = 0, -10 \leq x \leq 10, t = 1$. But a multi-soliton shape appears in Figure 4, with $a_0 = 0, k = 1, y = 1, z = 0, -10 \leq x \leq 10, t \leq 10$.

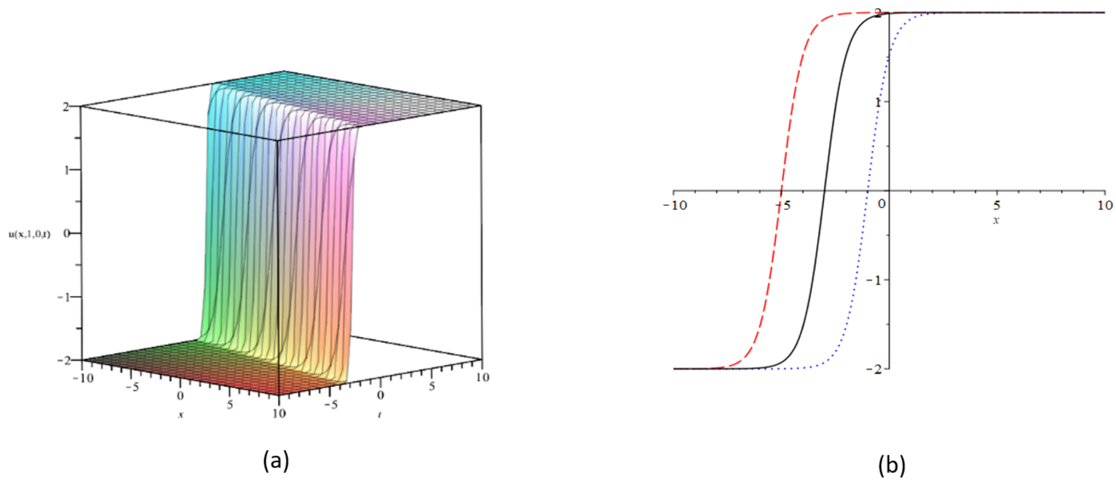


Figure 1: (a) Three-dim for $u_1(x, y, z, t)$ with $a_0 = 0, k = 1, y = 1, z = 0, -10 \leq x \leq 10, -10 \leq t \leq 10$, (b) two-dim for $u_1(x, y, z, t)$ with (red $t = 1$, blue $t = 0$, and black $t = 0.5$).

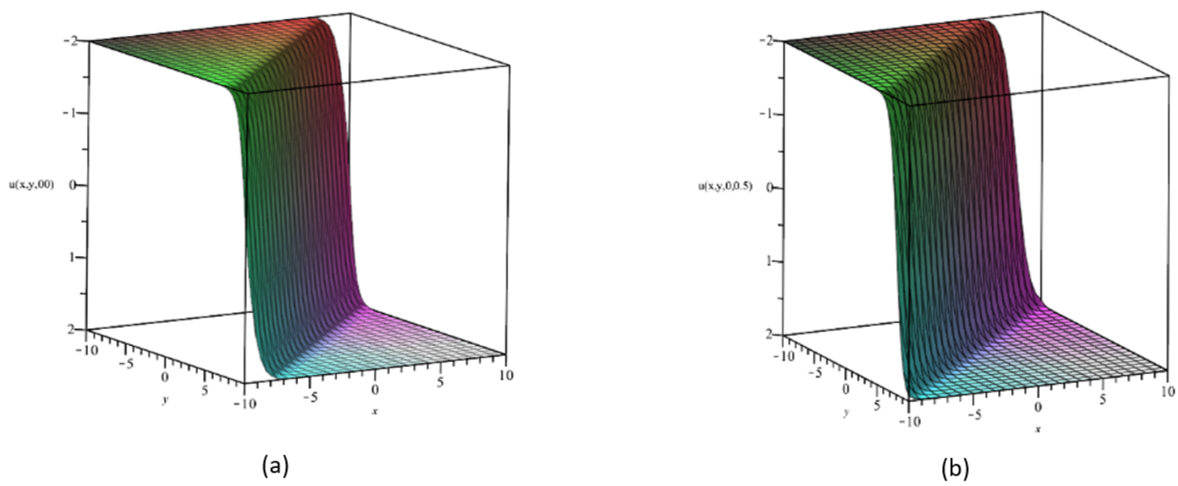
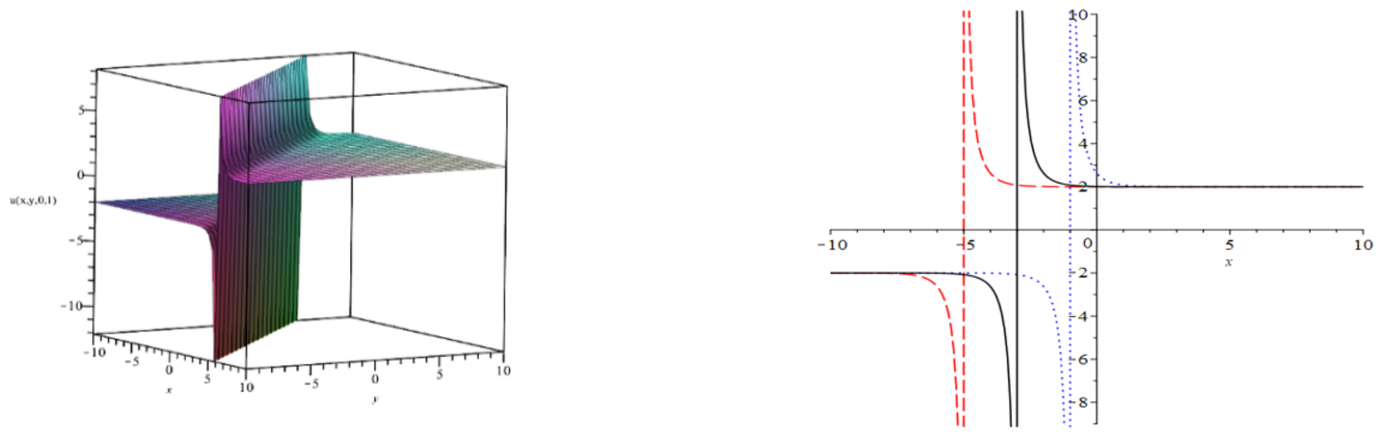


Figure 2: (a) Three-dim for $u_1(x, y, z, t)$ with $a_0 = 0, k = 1, -10 \leq y \leq 10$, $z = 0, -10 \leq x \leq 10, t = 0$, (b) Three-dim for $u_1(x, y, z, t)$ with $a_0 = 0, k = 1, -10 \leq y \leq 10, z = 0, -10 \leq x \leq 10, t = 0.5$



(a) (b)

Figure 3: (a) Three-dim for $u_2(x, y, z, t)$ with $a_0 = 0, k = 1, -10 \leq y \leq 10, z = 0, -10 \leq x \leq 10, t = 1$, (b) two-dim for $u_2(x, y, z, t)$ with (red $t = 1$, blue $t = 0$, and black $t = 0.5$).

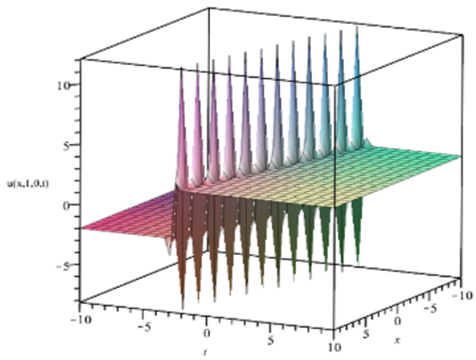


Figure 4: Three-dim for $u_2(x, y, z, t)$ with $a_0 = 0, k = 1, y = 1, z = 0, -10 \leq x \leq 10, t \leq 10$

Nursena et al., in [34], found the traveling wave solutions for the problem as kink-wave solution, multi soliton solution and Singular kink solution by using Bernoulli sub-ODE, $\frac{1}{G'}$ and modified Kudryashov methods.

Melih et al., in [35], found the traveling wave solutions for the problem as periodic-solution, rogue wave solutions and bright and dark wave solutions by using bilinear neural network method (BNNM).

Min-Jie Dong et al., in [36], found the traveling wave solutions for the problem as one-soliton solution, two-soliton solution, breather wave solutions and rogue wave solution by using the homoclinic test method.

In this article we present new approach for tanh and extended-tanh methods to solve the problem and found more general solution. The current study shows that suggested approaches is more powerful, efficient, and reliable for investigating different nonlinear evolution equations and also obtained results of type kink-wave solutions, Singular kink solution, and multi-soliton solution.

5. Discussion the results

The tanh technique is a highly efficient algebraic method utilized to find the different type of solution for non-linear wave equations. Malfliet in [35] proposed this method for calculating traveling wave solutions. Malfliet employed tanh approach, since the authors used tanh as a new changeable, because all derivatives of a tanh are supplied by the tanh itself. With the help of a symbolic arithmetic system like maple has been used to obtain the accurate solutions.

Suggested modification does not depend on derivatives of Eq. (7) but depend on derivatives of Eq. (18). So we get 1^{st} derivative only and can be determined from Eq. (20).

We note that the Eq. (14) matches the Eq. (28). On the other hand, while the modified extended method gives us three solution set, but the tanh method gives us one solution only.

6. Conclusions

In this article, we used the Tanh method and then suggest modified extended Tanh method to find the solutions of type travelling wave to the three-dimensional space new Hirota bilinear equation (NHBE). These solutions of type travelling wave are denoted by trigonometric and hyperbolic functions with arbitrary parameters. When these parameters are given as special values, single waves emerge from travelling waves. A comparison between methods used, one can see that the methods are effective, direct, and concise. These methods can also be used to solve many other non-linear equations.

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