Advances in the Theory of Nonlinear Analysis and its Applications 7 (2023) No. 4, 170-180.
https://doi.org/10.17762/atnaa.v7.i4.314
Available online at www.https://atnaea.org/
Research Article

Advances in the Theory of Nonlinear Analysis and its Applications
ISSN: 2587-2648
Peer-Reviewed Scientific Journal

Comparison Various Estimation Methods for $R = P(Y_1 < X < Y_2)$ Utilizing Restricted Generalized Weibull Distribution

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Abstract

The stress-strength S-S method employs a variety of estimation techniques, including maximum likelihood, shrinkage and least square, to determine and estimate the reliability of a particular system $R = P(Y_1 < X < Y_2)$ while the system contains a single component with strength X subject to two stresses, Y_1 and Y_2. With the consumption of Monte Carlo simulation and the statistical measurement Mean Squared Error (MSE), the various estimation methods have been evaluated according to the Restricted Generalized Weibull Distribution (RGWD), the stresses Y_1 and Y_2 and the strength X constitute independent, non-identical random variables in our S-S model.

Keywords: Restricted generalized Weibull distribution; Stress–strength reliability; Maximum likelihood method; Shrunken method; Least Squares method; Mean Squared Error.
2010 MSC: 62Exx, 62E10.

1. Introduction

The Stress-Strength S-S reliability model is still commonly used by presenters on quantum and applied mechanics. Determining the mechanical reliability of a component with strength Y subjected to stress X presents problems when evaluating the stress-strength (s -s) reliability, including a single component, as

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Received September 20, 2023; Accepted: November 29, 2023; Online: December 24, 2023.

demonstrated by ($R = P(Y < X)$).The component of this system will stop performing if and only if the stress ever exceeds the body's capacity; [1].

According to ($R = P(Y_1 < X < Y_2)$), this deals with the situation where the strength X should both be more than and less than the stress Y_1 and Y_2; a person's blood pressure at all times should be within these ranges , which were first introduced by Kotz in 2003; [2]. Using existing U- statistical techniques, Waegeman et al. in 2008 proposed an efficient algorithm for computing $P(X < Y < Z)$ and its variance; [3]. Wang et al. in 2013 ,under the assumption that the three samples were independent and disconnected, statistical inference for $R = P(Y_1 < X < Y_2)$ was performed using nonparametric estimates and the jackknife empirical likelihood; [4]. For the estimate of $R = (Y_1 < X < Y_2)$, Amal et al. in 2013 developed the ML estimator, MOM estimator, Mix. estimator, and the stresses Y_1, Y_2, and the strength X have distribution follow Weibull; [5]. When the stresses Y_1 and Y_2 and the strength X have inverse Rayleigh distribution, Raheem, S.H., Kalaf, B.A., and Salman, A.N. in 2021, compare the estimation of $R = (Y_1 < X < Y_2)$; [6].

The failure rate function of the Weibull distribution is monotonic, but it can also be growing or decreasing. The failure rate is frequently non-monotonic in systems that are more complex, such electronic systems. Typically, this occurs as a higher failure rate early (referred to as "wear-in") and late (referred to as "wear-out") in the component duration. This can also be referred to as a "bathtub-shaped" failure rate, or a "modified bathtub" (MBT) (or "roller coaster") -shaped failure rate if the failure rate limit at time zero is zero. A lot of the Weibull distribution's types have been driven by the desire of generating a bathtub-shaped failure rate function, which is useful in reliability engineering; [7].

It is possible to create generalized Weibull distributions in a variety of methods. Members of this family typically include the common Weibull model as a special case. The common techniques are briefly described in the sentences that follow the hazard rate h(t) of the Weibull distribution or a generalized Weibull model may be increased by a new parameter, or the Weibull random variable may be transformed (linear, inverse, or log) to generate a new generalized distribution, or by the competing risk approach (minimum of two or more Weibull variables), or by mixtures of two or more Weibull variables, mixtures of two or more generalized Weibull variables, mixing a Weibull distribution with a generalized Weibull distribution, etc.; [8] Regular distributions with bathtub-shaped, unimodal, and a wide range of monotone hazard rates make up the Generalized Weibull family, a Weibull extension created by including a second shape parameter. It can be used to model lifespan data from population, survival, and reliability studies, as well as different extreme value data, and to build isotopes for testing of the exponential composite hypothesis. The traditional Weibull model was proposed to be altered by Mudholkar and Srivastava in 1993 by the addition of a new parameter; [9]. Nassar and Eissa examined the characteristics of (GWD) in 2003; [10]. Pal and Woo compare the GWD with traditional Weibull and Gamma distributions based on the failure rate in 2006; [11].

The following presents the probability density function (PDF) of an r. v. X that is followed by GWD:

$$f\left(t;\alpha,\lambda\right) = \alpha\lambda t^{\lambda-1}e^{-t^{\lambda}}\left(1-e^{-t^{\lambda}}\right)^{\alpha-1} \qquad t;\alpha,\lambda > 0 \tag{1}$$

The cumulative distribution function (CDF) of X is also presented next:

$$F(t;\alpha,\ \lambda) = \left(1-e^{-t^{\lambda}}\right)^{\alpha} \qquad t;\alpha,\lambda > 0 \tag{2}$$

Wherever α refer to unidentified shape parameter and λ identified value denote to scale parameter. When the scale parameter λ is equal to 1, the distribution previously described will be known as the Restricted Generalized Weibul Distribution (RGWD).

The mentioned distribution was used to characterize failure time data and can be used to simulate a wide range of data sets (see Hahn and Shapiro, 1967). Relays, ball bearings, electronic parts, and even some businesses have had their lifespans approximated with this distribution. The significance of the bathtub curve in terms of hazards is frequently helpful to think about the function that indicates the chance of failure at the time when there haven't been any failures.

Assuming that the random variables Y_1, X, Y_2 follows the Restricted Generalized Weibul Distribution, the stress-strength model $R = (Y_1 < X < Y_2)$ is discussed in this study as an important component of the reliability system. In this context, various estimation strategies will be used to estimate the s-s reliability R of the Restricted Generalized Weibull distribution (RGWD) with Monte Carlo simulations built on the statistical criterion Mean Squared Error, the recommended estimators were compared.

A r. v. t that is followed by RGWD for $t > 0, \alpha > 0$ will have (PDF) and (CDF)

$$f(t;\alpha) = \alpha\ e^{-t}(1-e^{-t})^{\alpha-1}, \tag{3}$$

$$F(t;\alpha) = (1-e^{-t})^{\alpha} \tag{4}$$

At this point, if the RGWD with the relevant parameters is followed by the strength X and stresses Y_1 and Y_2, then:

$$f(x;\alpha_1) = \alpha_1\ e^{-x}(1-e^{-x})^{\alpha_1-1} \quad x;\alpha_1 > 0 \tag{5}$$

$$f(y1;\alpha_2) = \alpha_2 e^{-y1}(1-e^{-y1})^{\alpha_2-1} \quad y_1;\alpha_2 > 0, \tag{6}$$

$$f(y2;\alpha_3) = \alpha_3 e^{-y2}(1-e^{-y2})^{\alpha_3-1} \quad y_2;\ \alpha_3 > 0, \tag{7}$$

The (S-S) reliability is then described in the following ways; [5], and [6].

$$\begin{aligned} R &= P(Y_1 < X < Y_2) \\ &= P(X > Y_1) - P(X > Y_1, X > Y_2) \\ &\int_0^\infty F_{y1}(x) f(x)dx - \int_0^\infty F_{y1}(x) F_{y2}(x) f(x)\,dx \end{aligned}$$

Through switching Eqs. (3) and (4) w.r.t random variables and parameters, R becomes:

$$\begin{aligned} R &= \int_0^\infty \lambda\ e^{-y}(1-e^{-y})^{\lambda-1}\ dy \int_0^y \alpha\ e^{-x}(1-e^{-x})^{\alpha-1}dx \\ &= \int_0^\infty \lambda\ e^{-y}(1-e^{-y})^{\alpha+\lambda-1}dy \\ &= \frac{\alpha_1}{\alpha_1+\alpha_2} - \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3} \end{aligned}$$

$$R = \frac{\alpha_1\alpha_3}{(\alpha_1+\alpha_2)(\alpha_1+\alpha_2+\alpha_3)} \tag{8}$$

2. Techniques of estimation $R = P(Y_1 < X < Y_2)$

2.1. The maximum likelihood estimator (MLE):

The likelihood function of the experiential sample $x_1, x_2, \ldots, x_n$ assumed for RGED(α_1), $y_1, y_2, \ldots, y_m$ are experiential samples from RGWD(α_2) and $y_1, y_2, \ldots, y_k$ are experiential samples from RGWD(α_3)

$$\begin{aligned} l = L(\alpha_1, x) &= \prod_{i=1}^n f(x_i) \\ &= \prod_{i=1}^n \alpha_1 e^{-xi}(1-e^{-xi})^{\alpha_1-1} \end{aligned}$$

$$= {\alpha_1}^n\ e^{-\sum_{i=1}^n xi} \prod_{i=1}^n (1-e^{-xi})^{\alpha_1-1} \tag{9}$$

Ln taken to both sides yields

$$Ln(l) = nLn\alpha_1 - \sum_{i=1}^n xi + (\alpha_1-1)\sum_{i=1}^n Ln\left(1-e^{-xi}\right) \tag{10}$$

The derivative of $Ln(l)$ regard for α_1 and equal the result to zero is shown as below:

$$\frac{dLn(l)}{d\alpha_1} = \frac{n}{\alpha_1} + \sum_{i=1}^{n} Ln\left(1 - e^{-xi}\right) = 0 \tag{11}$$

Unidentified α_1 is estimated by the ML as follows:

$$\hat{\alpha}_{1MLE} = \frac{-n}{\sum_{i=1}^{n} Ln\left(1 - e^{-xi}\right)}, \tag{12}$$

Using similar procedures, $\hat{\alpha}_{2MLE}$ and $\hat{\alpha}_{3MLE}$ become:

$$\hat{\alpha}_{2MLE} = \frac{-m}{\sum_{j=1}^{m} Ln\left(1 - e^{-y1j}\right)} \text{ and } \hat{\alpha}_{3MLE} = \frac{-k}{\sum_{l=1}^{k} Ln\left(1 - e^{-y2l}\right)} \tag{13}$$

We note that $\hat{\alpha}_{i1MLE}$ and $\hat{\lambda}_{MLE}$ biased since $E(\hat{\alpha}_{iMLE}) = \frac{\mathcal{Z}\alpha i}{\mathcal{Z}-1}$ for $i = 1, 2, 3$ and $\mathcal{Z}$ mention to n, m and k.

Hence, $\hat{\alpha}_{iUb} = \frac{\mathcal{Z}-1}{\mathcal{Z}}\hat{\alpha}_{iMLE}$, determination the unbiased estimator of α_i; $i = 1, 2, 3$.

By substituting the results in (12) and (13) in Eq. (8), we obtain:

$$\hat{R}_{MLE} = \frac{\hat{\alpha}_{1MLE}\hat{\alpha}_{3MLE}}{\left(\hat{\alpha}_{1MLE} + \hat{\alpha}_{2MLE}\right)\left(\hat{\alpha}_{1MLE} + \hat{\alpha}_{2MLE} + \hat{\alpha}_{3MLE}\right)} \tag{14}$$

2.2. Shrinkage Estimation Method (Sh)

In this subsection, we consider Thompson's proposal in 1968 for a shrinkage estimation method, subject to prior knowledge which is very closed of the exact parameter's value from prior experiences or studies as initial value α_0 as well as the traditional estimator $\hat{\alpha}_{ub}$ using a factor as shrinkage weight $\Psi\left(\hat{\alpha}_{ub}\right), 0 \leq \Psi\left(\hat{\alpha}_{ub}\right) \leq 1$ as follows.

$$\hat{\alpha}_{sh} = \Psi\left(\hat{\alpha}_{ub}\right)\hat{\alpha}_{ub} + (1 - \Psi\left(\hat{\alpha}_{ub}\right))\alpha_0 \tag{15}$$

Where $(1 - \Psi\left(\hat{\alpha}_{ub}\right))$ signifies to trust of α_0 and $\Psi\left(\hat{\alpha}_{ub}\right)$ mentions to the belief of $\hat{\alpha}_{ub}$, which could be fixed value or depends on $\hat{\alpha}_{ub}$ or can be discovered by reducing the MSE for $\hat{\alpha}_{sh}$; see [12, 13, 14, 15].

In this section, we look at how sample sizes n, m and k affect the shrinkage weight factor in Eq. (15).

Accordingly, let's assume twice shrinkage weight factors $\Psi\left(\hat{\alpha}_{iub}\right) = \omega_i = \sqrt[3]{\left|\frac{\sin\mathcal{Z}}{\mathcal{Z}}\right|}$, and $\Psi\left(\hat{\alpha}_{iub}\right) = \zeta_i = \frac{\beta(\mathcal{Z}+1,1)}{\beta(\mathcal{Z},1)}$ for $i = 1, 2, 3$ w.r.t $\mathcal{Z}(n, mandk), \beta(.,.)$ refer to Beta function. At that time, two shrinkage estimators $\hat{\alpha}_{iSh}$ come to be

$$\hat{\alpha}_{i\ Sh1} = \omega_i\ \hat{\alpha}_{iUb} + (1 - \omega_i\ )\alpha_0, i = 1, 2, 3 \tag{16}$$

$$\hat{\alpha}_{i\ Sh2} = \zeta_i\hat{\alpha}_{iUb} + (1 - \zeta_i)\alpha_0, i = 1, 2, 3 \tag{17}$$

The agreeing (S-S) reliability employing shrinkage method $\hat{R}_{shj}$ will be

$$\hat{R}_{shj} = \frac{\hat{\alpha}_{1shj}\ \hat{\alpha}_{3\ shj}}{\left(\hat{\alpha}_{1\ shj} + \hat{\alpha}_{2\ shj}\right)\left(\hat{\alpha}_{1shj} + \hat{\alpha}_{2\ sh1} + \hat{\alpha}_{3\ shj}\right)}, j = 1, 2 \tag{18}$$

2.3. Least Squares Estimator Method (LS)

The Least Squares estimator, a technique utilized in numerous mathematics and engineering applications, is covered in this subsection.

The key idea is that it is possible to reduce the amount of squared errors between the imposed simple values and estimated value; [6].

$$S = \sum_{i=1}^{n} \left[F\left(x_i\right) - E\left(F(x_i)\right)\right]^2 \qquad i = 1, 2, 3, \ldots, n \tag{19}$$

For RGWED, we employ the $F(.)$ as below.

$$F(x_i) = (1 - e^{-xi})^{\alpha_1}$$

While, $E(F(x_i)) = P_i; P_i = \frac{i}{n+1}$, where $i = 1, 2, \ldots, n$

$$F(x_i) = E(F(x_i))$$

$$(1 - e^{-xi})^{\alpha_1} = \frac{i}{n+1}$$

Now, the logarithm is used on both sides

$$\alpha_1 Ln(1 - e^{-xi}) = LnP_i$$

$$\alpha_1 Ln(1 - e^{-xi}) - LnP_i = 0 \tag{20}$$

Eqs. (19) and (20) combined will so result in:

$$S = \sum_{i=1}^{n} \left[\alpha_1 Ln(1 - e^{-xi}) - LnP_i\right]$$

The partial derivatives of the natural logarithm are found to be equal to zero, and we obtain:

$$\frac{dS}{d\alpha_1} = 2\sum_{i=1}^{n} \left[\alpha_1 Ln(1 - e^{-xi}) \; - \; LnP_i \;\right] Ln(1 - e^{-xi})$$

$$\sum_{i=1}^{n} \left[\alpha_1 \; Ln\left(1 - e^{-xi}\right) \check{\ } LnPi\right] Ln\left(1 - e^{-xi}\right) = 0$$

$$\sum_{i=1}^{n} \alpha_1 Ln\left(1 - e^{-xi}\right) Ln\left(1 - e^{-xi}\right) = \sum_{i=1}^{n} \; LnP_i Ln\left(1 - e^{-xi}\right)$$

$$\hat{\alpha}_{1\ LS} \quad = \frac{\sum_{i=1}^{n} \; LnP_i Ln\left(1 - e^{-xi}\right)}{\left(\sum_{i=1}^{n} Ln\left(1 - e^{-xi}\right)\right)^2} \tag{21}$$

Similarly, as well

$$\hat{\alpha}_{2\ LS} \;= \frac{\sum_{j=1}^{m} \; LnP_j Ln\left(1 - e^{-yj}\right)}{\left(\sum_{j=1}^{m} Ln\left(1 - e^{-yj}\right)\right)^2}, P_j = \frac{j}{m+1}, j = 1, 2, \ldots, m \tag{22}$$

$$\hat{\alpha}_{3\ LS} = \frac{\sum_{l=1}^{n} \; LnP_l Ln\left(1 - e^{-zl}\right)}{\left(\sum_{l=1}^{k} Ln\left(1 - e^{-zl}\right)\right)^2}, P_{\ l} = \frac{l}{k+1}, l = 1, 2, \ldots, k$$

Applying $\hat{\alpha}_{1\ LS}, \hat{\alpha}_{2\ LS}$ and $\hat{\alpha}_{3\ LS}$ to Eq.(8) gives us approximately $\hat{R}_{LS}$:

$$\hat{R}_{LS} = \frac{\hat{\alpha}_{1ls}\hat{\alpha}_{3\ ls}}{(\hat{\alpha}_{1\ ls} + \hat{\alpha}_{2\ ls})\,(\hat{\alpha}_{1ls} + \hat{\alpha}_{2\ ls} + \hat{\alpha}_{3\ ls})} \tag{23}$$

3. Monte Carlo simulation technique

The proposed s-s reliability estimators were accomplished in section 2 using unlike sample sizes (30, 60, and 90), established on 1000 duplication via MSE criteria.

In this context, the statistical implications were calculated to compare the performance of the offered estimators. Following is how a Monte Carlo simulation was employed in this situation; [6, 16, 17].

Step1: Create a random sample that follows the uni $f(0, 1)$ as $u1, u2, \ldots, u_n, w_1, w_2, \ldots, w_m$ and $v_1, v_2, \ldots, v_k$.

Step2: Convert the values of random samples in step (1) to values follows RGWD in relation to the CDF; $F(x)$:

$$i.e.; F(x) = (1 - e^{-xi})^{\alpha_1}$$

$$ui = \ (1 - e^{-xi})^{\alpha_1}$$

$$x_i = -ln\left(1 - u_i{}^{\frac{1}{\alpha}}1\right), \quad i = 1, 2, \ldots, n$$

Using a similar strategy, y_j and z_l generates $y_j = -ln\left(1 - w_i{}^{\frac{1}{\alpha}}2\right)$ and $z_l = -ln\left(1 - v_l{}^{\frac{1}{\alpha}}3\right); j = 1, 2, \ldots, m$ and $l = 1, 2, \ldots, k$.

Step3: recall R from Eq.(8).

Step4: Use Eq.(14) to compute the MLE in R.

Step5: Calculate R's shrinkage estimator using Eq.(18).

Step6: Using Eq.(23), the R Least Squares estimator is used.

Step7: Founded on ($L = 1000$) duplication, evaluate the MSE,

It was become clear that: $MSE = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i - R\right)^2$. Any proposed estimators of the true value of R are denoted by $\hat{R}$. All of the expected outcomes are shown in the Tables 1 through 8 below:

Table 1: Shows the estimated values of R; $\alpha_1 = 2$ and $\alpha_2 = 2$, $\alpha_3 = 2$, R= 0.16667.

(n,m,k)	$\hat{R}_{MLE}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{LS}$
(30,30,30)	0.11154	0.14704	0.11334	0.1032
(30,60,30)	0.17473	0.16352	0.17302	0.25189
(30,90,30)	0.17765	0.17306	0.17379	0.32825
(60,30,30)	0.17455	0.16980	0.17466	0.14121
(60,60,30)	0.17781	0.17063	0.17567	0.25070
(60,90,30)	0.19428	0.17662	0.19054	0.34130
(90,30,30)	0.13939	0.15762	0.14068	0.12064
(90,60,30)	0.18383	0.17157	0.18151	0.22390
(90,90,30)	0.15916	0.16571	0.1567	0.26396
(30,30,60)	0.16514	0.16636	0.16705	0.10444
(30,60,90)	0.15527	0.16423	0.15547	0.12056
(30,90,90)	0.16294	0.16551	0.16235	0.15430
(60,30,90)	0.21565	0.18294	0.21690	0.10672
(60,60,90)	0.17693	0.16786	0.17726	0.12818
(90,60,90)	0.17670	0.16915	0.17736	0.13314
(90,90,90)	0.17857	0.16952	0.17844	0.18371

Table 2: Displays the MSE of $\hat{R}$; $\alpha_1 = 2$ and $\alpha_2 = 2$, $\alpha_3 = 2$, $R = 0.16667$.

(n,m,k)	MLE	Sh_1	Sh_2	LS	Best
(30,30,30)	3.0393e-07	3.8532e-08	2.8439e-07	4.0275e-07	Sh1
(30,60,30)	6.5031e-09	9.873e-10	4.0314e-09	7.2631e-07	Sh1
(30,90,30)	1.2058e-08	4.0858e-09	5.0793e-09	2.6109e-06	Sh1
(60,30,30)	6.2202e-09	9.7941e-10	6.3958e-09	6.4806e-08	Sh1
(60,60,30)	1.2417e-08	1.5724e-09	8.113e-09	7.0613e-07	Sh1
(60,90,30)	7.6225e-08	9.9128e-09	5.7012e-08	3.0495e-06	Sh1
(90,30,30)	7.4423e-08	8.1804e-09	6.7526e-08	2.1184e-07	Sh1
(90,60,30)	2.9445e-08	2.4034e-09	2.204e-08	3.2758e-07	Sh1
(90,90,30)	5.6356e-09	9.2127e-11	9.925e-09	9.4654e-07	Sh1
(30,30,60)	2.3271e-10	9.1032e-12	1.4946e-11	3.8722e-07	Sh1
(30,60,90)	1.2993e-08	5.9301e-10	1.2537e-08	2.1258e-07	Sh1
(30,90,90)	1.3891e-09	1.3308e-10	1.8655e-09	1.5296e-08	Sh1
(60,30,90)	2.3994e-07	2.6483e-08	2.5236e-07	3.5935e-07	Sh1
(60,60,90)	1.0533e-08	1.4193e-10	1.1212e-08	1.4812e-07	Sh1
(90,60,90)	1.0068e-08	6.1679e-10	1.1425e-08	1.1243e-07	Sh1
(90,90,90)	1.4179e-08	8.1532e-10	1.385e-08	2.9033e-08	Sh1

Table 3: Shows the estimated values of R; $\alpha_1 = 1.5$ and $\alpha_2 = 2$, $\alpha_3 = 2$, $R= 0.15584$.

(n,m,k)	$\hat{R}_{MLE}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{LS}$
(30,30,30)	0.16988	0.16101	0.16945	0.18768
(30,60,30)	0.12558	0.14710	0.12419	0.21313
(30,90,30)	0.17036	0.16347	0.16601	0.34803
(60,30,30)	0.16142	0.15806	0.16185	0.12863
(60,60,30)	0.18785	0.16078	0.18553	0.29626
(60,90,30)	0.14759	0.15164	0.14573	0.30249
(90,30,30)	0.18306	0.16497	0.18293	0.11517
(90,60,30)	0.12636	0.1453	0.12622	0.16916
(90,90,30)	0.15235	0.15481	0.15026	0.24652
(30,30,60)	0.17148	0.16172	0.17261	0.12764
(30,60,90)	0.14035	0.15288	0.14025	0.11446
(30,90,90)	0.14200	0.15169	0.14108	0.16432
(60,30,90)	0.12974	0.14590	0.13329	0.052855
(60,60,90)	0.17204	0.15898	0.17232	0.11979
(90,60,90)	0.20339	0.16385	0.20330	0.13634
(90,90,90)	0.16999	0.15966	0.16985	0.16393

Table 4: Displays the MSE of $\hat{R}$; $\alpha_1 = 1.5$ and $\alpha_2 = 2$, $\alpha_3 = 2$, $R = 0.15584$.

(n,m,k)	MLE	Sh_1	Sh_2	LS	Best
(30,30,30)	1.9697e-08	2.6733e-09	1.8514e-08	1.0136e-07	Sh1
(30,60,30)	9.1579e-08	7.6378e-09	1.0017e-07	3.2821e-07	Sh1
(30,90,30)	2.1075e-08	5.8145e-09	1.0336e-08	3.6936e-06	Sh1
(60,30,30)	3.111e-09	4.9309e-10	3.6114e-09	7.4052e-08	Sh1
(60,60,30)	1.0244e-07	2.4403e-09	8.8152e-08	1.9717e-06	Sh1
(60,90,30)	6.8194e-09	1.771e-09	1.0233e-08	2.1504e-06	Sh1
(90,30,30)	7.4046e-08	8.3345e-09	7.3369e-08	1.6546e-07	Sh1
(90,60,30)	8.6933e-08	1.1112e-08	8.7757e-08	1.7721e-08	Sh1
(90,90,30)	1.2175e-09	1.0726e-10	3.1234e-09	8.2222e-07	Sh1
(30,30,60)	2.4446e-08	3.4557e-09	2.812e-08	7.9521e-08	Sh1
(30,60,90)	2.3992e-08	8.7925e-10	2.4309e-08	1.713e-07	Sh1
(30,90,90)	1.9161e-08	1.7254e-09	2.1799e-08	7.1769e-09	Sh1
(60,30,90)	6.8166e-08	9.8832e-09	5.0875e-08	1.0607e-06	Sh1
(60,60,90)	2.6223e-08	9.8528e-10	2.7139e-08	1.2997e-07	Sh1
(90,60,90)	2.2604e-07	6.4099e-09	2.2521e-07	3.8037e-08	Sh1
(90,90,90)	2e-08	1.4563e-09	1.961e-08	6.5305e-09	Sh1

Table 5: Shows the estimated values of R; $\alpha_1 = 2$ and $\alpha_2 = 1.5$, $\alpha_3 = 3$, R= 0.26374.

(n,m,k)	$\hat{R}_{MLE}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{LS}$
(30,30,30)	0.25104	0.26112	0.25171	0.25310
(30,60,30)	0.24721	0.25967	0.24458	0.37127
(30,90,30)	0.15655	0.22930	0.15598	0.31663
(60,30,30)	0.27373	0.26799	0.2736	0.24285
(60,60,30)	0.29593	0.26797	0.29302	0.39242
(60,90,30)	0.24507	0.25769	0.24257	0.38434
(90,30,30)	0.27407	0.26796	0.27404	0.22199
(90,60,30)	0.25537	0.25942	0.25352	0.32121
(90,90,30)	0.29838	0.27394	0.29427	0.43292
(30,30,60)	0.24816	0.25721	0.25104	0.18017
(30,60,90)	0.32621	0.27710	0.32570	0.24243
(30,90,90)	0.25897	0.26288	0.2584	0.22148
(60,30,90)	0.21618	0.24830	0.22083	0.10822
(60,60,90)	0.25005	0.26051	0.25092	0.21284
(90,60,90)	0.24859	0.26119	0.24978	0.18199
(90,90,90)	0.27986	0.26860	0.27971	0.27180

Table 6: Displays the MSE of $\hat{R}$; $\alpha_1 = 2$ and $\alpha_2 = 1.5$, $\alpha_3 = 3$, $R = 0.26374$.

(n,m,k)	MLE	Sh_1	Sh_2	LS	Best
(30,30,30)	1.6124e-08	6.8253e-10	1.4453e-08	1.1316e-08	Sh1
(30,60,30)	2.7306e-08	1.6504e-09	3.669e-08	1.1565e-06	Sh1
(30,90,30)	1.1489e-06	1.1859e-07	1.1611e-06	2.7975e-07	Sh1
(60,30,30)	9.9819e-09	1.81e-09	9.7202e-09	4.3635e-08	Sh1
(60,60,30)	1.0362e-07	1.7891e-09	8.574e-08	1.6559e-06	Sh1
(60,90,30)	3.4828e-08	3.6498e-09	4.4808e-08	1.4546e-06	Sh1
(90,30,30)	1.0682e-08	1.7879e-09	1.0623e-08	1.7428e-07	Sh1
(90,60,30)	7.0015e-09	1.8661e-09	1.0429e-08	3.3038e-07	Sh1
(90,90,30)	1.2e-07	1.0412e-08	9.3209e-08	2.8622e-06	Sh1
(30,30,60)	2.426e-08	4.2581e-09	1.6109e-08	6.9835e-07	Sh1
(30,60,90)	3.9025e-07	1.7851e-08	3.839e-07	4.541e-08	Sh1
(30,90,90)	2.2758e-09	7.2601e-11	2.8452e-09	1.7853e-07	Sh1
(60,30,90)	2.2615e-07	2.3818e-08	1.8411e-07	2.4186e-06	Sh1
(60,60,90)	1.8718e-08	1.038e-09	1.6414e-08	2.5909e-07	Sh1
(90,60,90)	2.2949e-08	6.4655e-10	1.948e-08	6.6825e-07	Sh1
(90,90,90)	2.5983e-08	2.3641e-09	2.5507e-08	6.5059e-09	Sh1

Table 7: Shows the estimated values of R; $\alpha_1 = 3$ and $\alpha_2 = 2.5$, $\alpha_3 = 3.5$, $R = 0.21212$.

(n,m,k)	$\hat{R}_{MLE}$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{LS}$
(30,30,30)	0.22838	0.21845	0.22791	0.2219
(30,60,30)	0.17932	0.20061	0.17869	0.26222
(30,90,30)	0.15552	0.19362	0.15433	0.29624
(60,30,30)	0.16901	0.19893	0.1708	0.1587
(60,60,30)	0.24285	0.21833	0.23991	0.33926
(60,90,30)	0.22279	0.21411	0.21953	0.37613
(90,30,30)	0.23897	0.22250	0.23892	0.17301
(90,60,30)	0.22915	0.21631	0.2266	0.27503
(90,90,30)	0.26591	0.22833	0.26139	0.42365
(30,30,60)	0.20625	0.21336	0.20834	0.12534
(30,60,90)	0.15527	0.19849	0.15618	0.12452
(30,90,90)	0.21120	0.21182	0.21070	0.18494
(60,30,90)	0.20677	0.21068	0.21047	0.083682
(60,60,90)	0.20070	0.21012	0.20158	0.14533
(90,60,90)	0.23813	0.21761	0.23856	0.17953
(90,90,90)	0.19943	0.20922	0.19958	0.19447

Table 8: Displays the MSE of $\hat{R}$; $\alpha_1 = 3$ and $\alpha_2 = 2.5$, $\alpha_3 = 3.5$, $R = 0.21212$.

(n,m,k)	MLE	Sh_1	Sh_2	LS	Best
(30,30,30)	2.6434e-08	4.0116e-09	2.4929e-08	9.5663e-09	Sh1
(30,60,30)	1.0757e-07	1.3245e-08	1.1176e-07	2.5101e-07	Sh1
(30,90,30)	3.2041e-07	3.4216e-08	3.34e-07	7.0763e-07	Sh1
(60,30,30)	1.8583e-07	1.7388e-08	1.7074e-07	2.8542e-07	Sh1
(60,60,30)	9.4425e-08	3.8589e-09	7.722e-08	1.6165e-06	Sh1
(60,90,30)	1.1379e-08	3.9476e-10	5.483e-09	2.6899e-06	Sh1
(90,30,30)	7.209e-08	1.0777e-08	7.1821e-08	1.53e-07	Sh1
(90,60,30)	2.9012e-08	1.7582e-09	2.0978e-08	3.9574e-07	Sh1
(90,90,30)	2.8932e-07	2.6276e-08	2.4276e-07	4.4745e-06	Sh1
(30,30,60)	3.451e-09	1.5228e-10	1.43e-09	7.5314e-07	Sh1
(30,60,90)	3.232e-07	1.8569e-08	3.129e-07	7.6738e-07	Sh1
(30,90,90)	8.5079e-11	9.1605e-12	2.017e-10	7.3857e-08	Sh1
(60,30,90)	2.8636e-09	2.0648e-10	2.7429e-10	1.6497e-06	Sh1
(60,60,90)	1.3052e-08	4.0121e-10	1.1121e-08	4.4615e-07	Sh1
(90,60,90)	6.7658e-08	3.0151e-09	6.9923e-08	1.0623e-07	Sh1
(90,90,90)	1.6107e-08	8.4161e-10	1.5728e-08	3.1143e-08	Sh1

4. Results and numerical outcomes

In order to estimate reliability R, the least Mean Square Error (MSE) should be computed in the single stress-strength model for the Restricted Generalized Exponential Distribution RGED for all $n, m, k = (30, 60, 90)$ and the suggested estimators will be compared. This demonstrates that the shrinkage estimator $\hat{R}_{Sh1}$ is the best when using the prescribed shrinkage weight factors. Sometimes observe a vibration and disproportion between $\hat{R}_{MLE}$, $\hat{R}_{Sh2}$ and $\hat{R}_{LS}$ as well; these three estimators have always come in second best rank. Additionally, the least squares estimation method in most cases $\hat{R}_{LS}$ is now regarded as the latest estimator rank.

5. Conclusion

In terms of MSE, the suggested estimate technique $\hat{R}_{Sh1}$ outperforms the competition and ranks as the most reliable estimator. It is built on an unbiased estimator and a prior estimate that was constructed on the authors' experiences employing the shrinkage weight factor as a function of sample sizes. We recommended employing this type of estimating technique to estimate the parameters and reliability function as well as the stress-strength reliability model with various distributions.

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