

Figure 2: The concern graph show the disease free dynamics of $x(t)$ for various value of initial sizes and parameters values, $\Pi = 0.6$, $\alpha = 0.0001$, $\gamma_0 = 0.01$, $\mu_0 = 0.05$, $v = 0.01$, $\beta = 0.1$, $r = 0.5$, $s = 0.6$, $\mu_1 = 0.02$, $\sigma = 0.4$:

Figure 3: The graph show the time dynamics $y(t)$ around disease free point and various population sizes while value of epidemic parameters are $\Pi = 0.6$, $\alpha = 0.0001$, $\gamma_0 = 0.01$, $\mu_0 = 0.05$, $v = 0.01$, $r = 0.5$, $\beta = 0.1$, $\mu_1 = 0.02$, $\sigma = 0.6$, $\sigma = 0.4$:

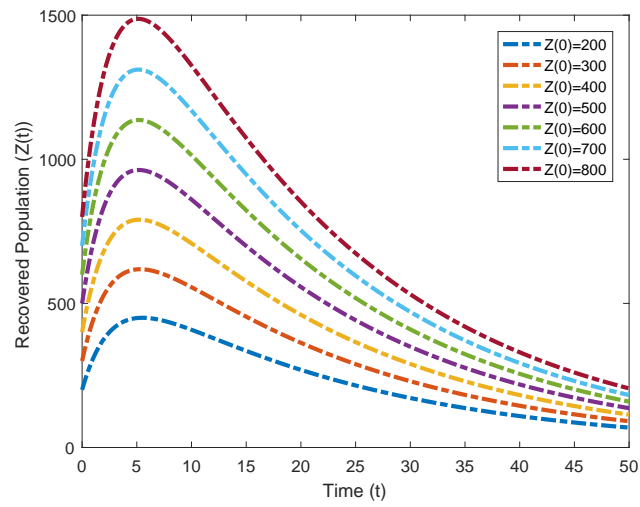


Figure 4: The graph visualizes the temporal dynamics $z(t)$ at disease free state and different initial conditions. Moreover, the epidemic parameters values are as: $\Pi = 0.6, \alpha = 0.0001, \gamma_0 = 0.01, \mu_0 = 0.05, v = 0.01, r = 0.5, \gamma_2 = 0.4, \beta = 0.1, s = 0.6, \mu_1 = 0.02$.

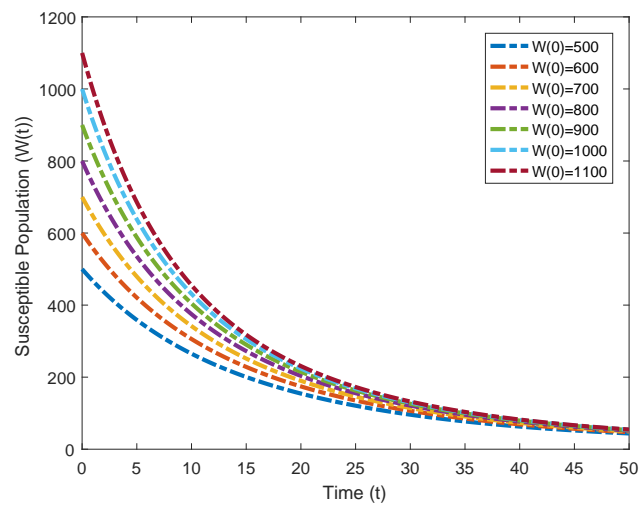


Figure 5: The graph visualizes the disease endemic dynamics of $w(t)$ for different value of initial sizes and parameters values, $\Pi = 0.8, \alpha = 0.00005, \gamma_0 = 0.01, \mu_0 = 0.05, \beta = 0.001, v = 0.001, r = 0.5, s = 0.2, \gamma_2 = 0.02, \mu_1 = 0.006$.

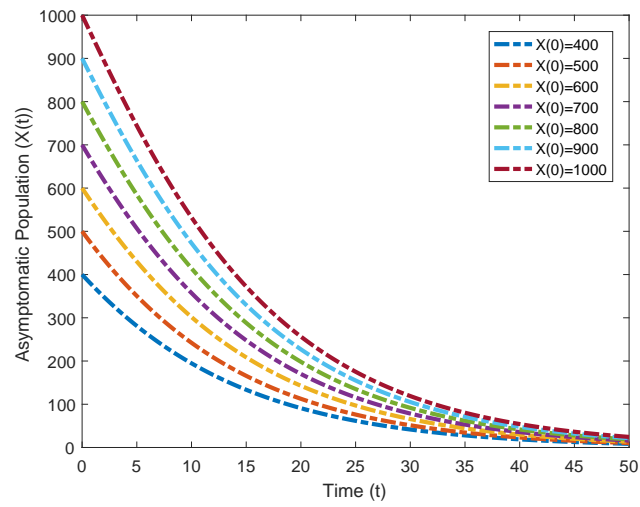


Figure 6: The graph demonstrate the dynamics $x(t)$ around endemic point and various population sizes while value of epidemic parameters are $\Pi = 0.8$, $\alpha = 0.00005$, $\gamma_0 = 0.01$, $\mu_0 = 0.05$, $\beta = 0.001$, $v = 0.001$, $r = 0.5$, $s = 0.2$, $\gamma_2 = 0.02$, $\mu_1 = 0.006$.

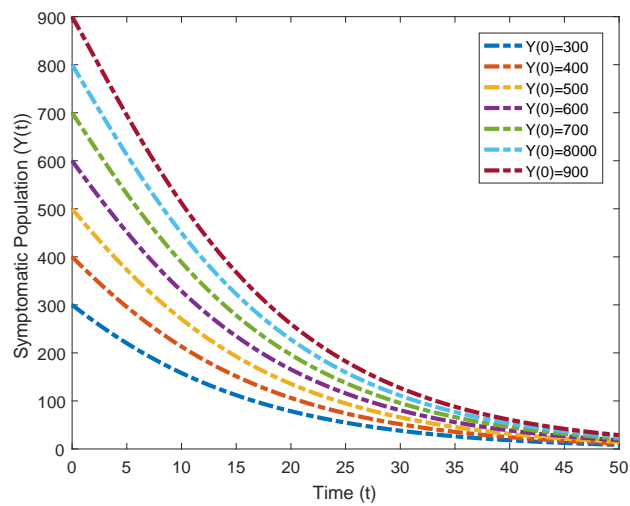


Figure 7: The graph demonstrate the temporal dynamics $y(t)$ at endemic state and different initial conditions. Moreover, the epidemic parameters values are as: $\Pi = 0.8$, $\alpha = 0.00005$, $\gamma_0 = 0.01$, $\mu_0 = 0.05$, $\beta = 0.001$, $v = 0.001$, $r = 0.5$, $s = 0.2$, $\gamma_2 = 0.02$, $\mu_1 = 0.006$.

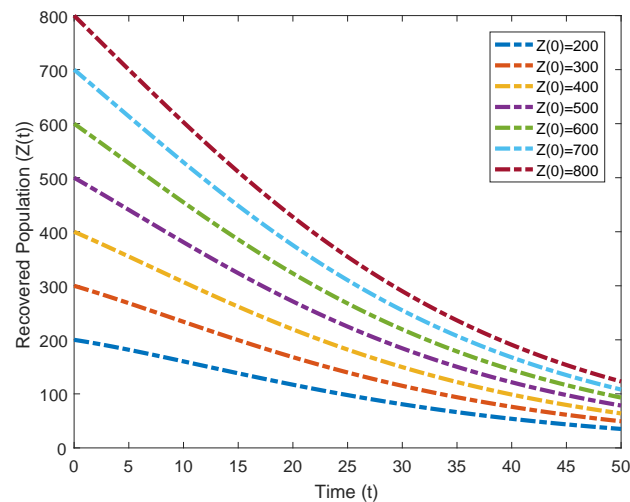


Figure 8: The graph demonstrate the temporal dynamics $z(t)$ at endemic state and different initial conditions. Moreover, the epidemic parameters values are as: $\Pi = 0.8$, $\alpha = 0.00005$, $\gamma_0 = 0.01$, $\mu_0 = 0.05$, $\beta = 0.001$, $v = 0.001$, $r = 0.5$, $s = 0.2$, $\gamma_2 = 0.02$, $\mu_1 = 0.006$.

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Declarations

- Ethical Approval: Not applicable.
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